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SATELLITE ORBIT DETERMINATION AND PREDICTION UTILIZING JPL GOLDSTONE 85-FT ANTENNA AND THE JPL TRACKING PROGRAM

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SATELLITE ORBIT DETERMINATION AND PREDICTION UTILIZING JPL GOLDSTONE 85-FT ANTENNA AND THE JPL TRACKING PROGRAM[1]

I. INTRODUCTION

As a part of the research and development effort in the area of tracking and communications, 108-mc tracking equipment was designed and built for the Goldstone 85-ft parabolic antenna. This system has been utilized to track several satellites. In particular, the satellite 1958 Beta II has been tracked many times, since it served as an excellent reference for the determination of systematic errors in the antenna system, e.g., servo errors and boresight errors. In each case, prediction information was supplied by the Goddard Space Flight Center, Washington D.C. Definitive orbital information was also supplied by that agency for purposes of comparison. This work has been extended to the computation of the orbit itself, utilizing the JPL lunar tracking program.

It should be emphasized that the Goldstone antenna was designed for operation at frequencies much higher than 108 mc. The beamwidth at this frequency is 8 deg. A theoretical rms angle error greater than 0.1 deg should be expected at the signal levels available from 1958 Beta II.

The orbit determination and prediction procedures should be considered as an integral part of the radio tracking system. This tracking program was described in detail in the previous paper. It is of interest to compute the orbit for the following reasons:

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[1]This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, underContract No. NASw-6, sponsored by the National Aeronautics and Space Administration.

1. To determine the feasibility of orbit determination from one station with one pass, two successive passes, and three successive passes.
2. To study the effects of correlated errors in the angle information due to antenna deformation, wind, etc., on orbit determination and to acquire some insight into the removal of such errors.
3. To study the application of the JPL lunar tracking program to the satellite orbit determination problem.
4. To study the problem of orbit determination in real time on a pass-to-pass basis.
5. To determine the accuracy with which predictions can be computed from an orbit so determined.
6. To study the effect of combining angle observations with doppler velocity observations and with range observations.
7. To study the combination of angle data and velocity or range data in the orbit program and to determine the required accuracy of doppler and range measurement systems.
8. To compare the results of satellite orbit computations to the results of lunar tracking experiments such as Pioneer IV so as to better determine the over-all specifications and design changes of the tracking net.
9. To add to the general understanding of the JPL tracking program.

The JPL tracking program is essentially a least squares fitting of observations to the equations of motion. At a specified time injection coordinates, which include position of the probe and magnitude and direction of the velocity vector, are computed from the observations. The Cowell's integration was used to predict the station coordinates at any future time. The program was able to use angle, range, and range rate observations from several stations. The refraction corrections are made utilizing a standard atmosphere. Known systematic errors are removed with appropriate biasing, etc.

II. ORBIT DETERMINATION FROM ANGLE OBSERVATIONS ONLY

Three successive passes of 1958 Beta II were tracked at Goldstone on April 24, 1959. These observations were selected for orbit computation. The portions of the orbit covered by each successive pass are indicated in Fig. 1. The wide bar indicates the first pass; the medium-size bar indicates the coverage for the second pass, and the smaller bar indicates the coverage for the third pass. It can be seen from this that about a fourth of the orbit was covered with the three passes. The local hour angle and the declination angle of the satellite were measured from horizon to horizon for each pass. A time at the beginning of the first pass over Goldstone was selected as the injection epoch for all the computations. The injection conditions determined for each case were integrated for about 4 hours; this corresponded to approximately 2 complete revolutions of the satellite about the Earth. The local hour angle and declination angle were computed from this integration at selected times when the satellite was visible over Goldstone, i.e., times during the third pass. These coordinates were compared with true coordinates supplied by the Goddard Space Flight Center from the orbit determined from the Minitrack System. This reference orbit was considered to be accurate to about 1 min of arc for the 1958 Beta II.

An orbit was determined with the first pass observations only (414 points). The resulting orbit fit the data with an rms error in hour angle of 0.268 deg and 0.160 deg in declination angle. These numbers are due mainly to the noise on the raw data. The orbit so determined was poor, and the comparison with the reference orbit indicated the probe's position had slipped about 12 min by the time of the third pass. This orbit would be nearly useless for prediction purposes. That the poor orbit is primarily due to the geometry of the problem and the use of angle observations alone and not to noise on the data will be shown below.

The orbit was then determined by combining the first- and second-pass angular data (about 700 points). The injection conditions so computed were then used to predict points during the third pass. The true hour angle and declination angle from the definitive orbit are presented in Fig. 2. Deviations of the predictions in local hour angle from the true trajectory are presented in Fig. 3 along with the predictions generated with all three passes of angular observations (above 1000 points). The deviations of the predictions in declination angle are presented in Fig. 4. These figures indicate that the prediction is accurate to about 0.1 deg or better at the horizon. It can be seen from these curves that excellent predictions can be made on a relatively short time basis from just two passes. The improvement in going to three passes is not remarkably significant. The shape of the curve is due to the complex geometry of the problem. The errors are considerably larger at the middle of the pass because of the angular rates; i.e., the error in the orbits is primarily a time slippage (error in period) of the probe along the orbital path. For these orbits, a slippage of about 1 sec would explain most of the error. The rms deviations of the data for the two-pass orbits are approximately 0.27 deg in local hour angle and 0.35 deg in declination angle. For the third pass, the rms errors are 0.18 deg in local hour angle and 0.29 deg in declination angle. These results seem to indicate that, within reasonable limits, the primary factor in the orbit determination is the geometry and not the noise on the data. The prediction error in slant range for the two cases is presented in Fig. 5 for completeness.

It can be concluded from the above analysis that highly accurate predictions can be generated from the orbit determined with two successive passes over the one station with angle data alone. It further shows that a poor orbit is found with one pass of angle data alone, and experience with these computations indicates that this is quite unrelated to the rms noise on the observations. It should be pointed out that bias errors or other systematic errors in the observations would have a very serious effect on an orbit determined with one pass.

III. ORBIT DETERMINATION FROM ANGLE OBSERVATIONS COMBINED WITH RANGE OR DOPPLER OBSERVATIONS

The first-pass observations were selected to study systematically the improvement in one-pass-one-station orbit determination by combining slant range or slant-range rates with angle data. Velocity data accurate to 0.01 m/sec was artificially generated with the three-pass orbit injection conditions and added to the angle data. The orbit determined this way probably represents the limit using this particular set of angle data. Orbits were also computed using velocity accurate to 1 m/sec and accurate to 10 m/sec combined with the angular observations. The results are presented in Figs. 6-8. Figure 6 shows the error in hour-angle predictions for the three cases. Figure 7 shows the prediction error in the declination angle for the three cases. The prediction error in slant range for two of the three cases is presented in Fig. 8. The data indicate that velocity observations accurate to 1 m/sec or better combined with the particular angular data available gave predictions for this pass better than 1 deg at the beginning of the pass. This is not an unreasonable angle to search for acquisition purposes. However, the probe must be acquired near the horizon or the errors get very large.

A similar analysis was done with the slant-range data good to the nearest 100 meters and to the nearest 1000 meters. These results are presented in Figs. 9-11. Figure 9 presents the error in the hour angle prediction for the two cases. Figure 10 shows the declination angle prediction error for the two cases. It can be seen that for the errors present in the angle observations there is no significant difference in the two cases. They both serve to give a much improved orbit over that determined by angular observations alone. Apparently, for this particular case, range data accurate to 1 km is nearly equivalent to velocity data accurate to 0.01 m/sec. This graphically indicates the importance of range measuring systems in orbit determination. Prediction error in the slant range for the two cases is presented in Fig. 11 for completeness.

In summary, the above analysis indicates that rather excellent predictions over several hours can be computed with angular data for one pass as bad as 0.2 deg rms when combined with velocity data better than 1 m/sec or range data better than 1 km.

IV. ORBITAL ELEMENTS

It is very useful in studying the results reported above to consider the orbital elements of the osculating ellipse at a specific epoch. It was convenient computationally to select an epoch at the beginning of the third pass. First of all, it was necessary to compute the orbital elements of the reference orbit at this epoch. It was necessary to use the tracking program and the data from Fig. 2. Range information to the nearest kilometer was also available. This combination of data resulted in an orbit with an rms error of 0.021 deg in hour angle, 0.005 deg in declination angle, and 260 meters rms in range. This is an excellent fit, but the remarks above concerning orbit determination from one pass certainly apply to this case. Therefore, the orbit so determined cannot be considered as a "true" orbit. This was apparently the best set of orbital elements available to the author.

The orbital elements for the orbits determined from angular observations alone are presented in Table 1. In the first row are the elements computed from the reference orbit. Computations of the probable error for these elements were made from the rms deviations reported above. The orbital elements computed for the orbit from two passes and from three passes are in excellent agreement with the reference orbit. However, it should be noted that the elements from the two-pass orbit and the three-pass orbit are in much closer agreement than with the reference itself. This suggests to me that the computed orbital elements are better than the true elements, as I have defined them. It should be noted that the period computed for the one-pass orbit is high by about 6 min, which essentially explains the time slippage in the prediction data for this case reported above. Also included in the last two columns of this table are the rms deviations of the angular observations from the fit. It can be seen from these that the quality of the orbits is apparently unrelated to the relative sizes of these deviations. This is probably because the rms deviations from the true orbit are basically noise.

Table 2 presents the elements for the one-pass orbits computed from angle plus velocity accurate to 0.01, 1, and 10 m/sec. The three-pass orbit was selected as the reference orbit in this case because the velocity information was generated from the three-pass orbit injection conditions. Table 3 presents the elements for the one-pass range data orbit. These orbital elements are in excellent agreement with the reference orbital elements, reiterating the value of the range information to the orbit-determinating problem.

The above study has been limited to one orbit, of course; and, in particular, all the comparisons were limited to one pass of this one orbit. Future study is required to learn the effects of going to orbits of different eccentricities and semimajor axes. This study will be considered as a foundation on which a theoretical study may be based.

V. CONCLUSIONS

The satellite tracking and orbit-determination experiments have yielded the following results:

1. The analysis shows that highly accurate predictions can be generated from the orbit determined with two successive passes over one station with angular data alone. It shows that a poor orbit is found with one pass of angular data alone, which is due more to the geometry involved than to the rms error on the data.
2. Analysis indicates that excellent predictions can be computed with angular data from one pass as bad as 0.2 deg rms when combined with velocity data better than 1 m/sec or range data better than 1 km for this particular orbit.
3. Further study is required to determine the design specifications for future doppler and range measurement systems.
4. Experience with the computations indicates that sufficiently accurate predictions can be generated for acquisition purposes for satellites, with one pass from one station in real time if an independent measurement of doppler velocity or range is available.
5. The effects of correlated errors due to the deformations of the antenna structure, etc., were found to be negligible in the orbit determination.
6. The orbit for 1958 Beta II was essentially determined with three passes.

NOMENCLATURE

a = semimajor axis in Earth radii

e = eccentricity

i = inclination, deg

ω = argument of perigee, deg

Ω = longitude of the node, deg

M = mean anomaly at epoch, deg

P = period, min

σ_α = standard deviation of hour angle from the fit, deg

σ_δ = standard deviation of declination angle from the fit, deg

$\sigma_{\dot{r}}$ = standard deviation of doppler velocity from fit, m/sec

σ_r = standard deviation of slant range from fit, meters

Table 1. Osculating Orbital Elements for the Satellite 1958 Beta II

Epoch: 24 Apr, 1 08 00 GMT

Source of Data	a	e	i	ω	Ω	M	P	σ_α	σ_δ
GSFC definitive angles and range	1.36056 ±0.00013	0.190766 ±0.000269	34.2304 ±0.00001	106.028 ±0.023	16.596 ±0.023	-23.354	134.120 ±0.019	0.0206	0.0048
Observations, one pass	1.40187	0.206287	34.2775	100.4161	17.6660	-49.0074	140.243	0.2680	0.1604
Observations, two passes	1.36067	0.190439	34.2633	106.1067	17.3606	-22.6062	134.1064	0.3521	0.2727
Observations, three passes	1.36066	0.190502	34.2272	106.1905	17.2785	-22.6018	134.1043	0.2923	0.1845

Table 2. Osculating Orbital Elements From One-Pass Angle Observations and Computed Doppler Data

Epoch: 24 Apr, 1 08 00 GMT

Source of Data	a	e	i	ω	Ω	M	P	σ_α	σ_δ	$\dot{r}$
Reference orbit from 3 passes	1.36066	0.190502	34.2272	106.1905	17.278	-22.6018	134.1043	0.2923	0.1845	-
$\dot{r}$ accurate to nearest 0.01 m/sec	1.36041	0.190435	34.2329	106.201	17.296	-22.4249	134.068	0.2662	0.1600	0.002
$\dot{r}$ accurate to nearest 1.0 m/sec	1.35990	0.190215	34.2382	106.262	17.290	-22.080	133.9927	0.2654	0.1643	0.288
$\dot{r}$ accurate to nearest 10.0 m/sec	1.35822	0.190170	34.2378	106.651	17.271	-21.015	133.7433	0.2634	0.1642	0.260

Table 3. Osculating Orbital Elements From One-Pass Angle Observations and Computed Slant Range Data

Epoch: 24 Apr, 1 08 00 GMT

Source of Data	a	e	i	ω	Ω	M	P	σ_α	σ_δ	σ_r
Reference orbit 3 passes angle data	1.36066	0.190502	34.2272	106.1905	17.2785	-22.6018	134.1043	0.2923	0.1845	-
r to nearest 100 meters	1.360413	0.190436	34.2334	106.204	17.294	-22.428	134.0679	0.2652	0.1642	33.4
r to nearest 1000 meters	1.360447	0.190954	34.2216	106.333	17.282	-22.498	134.073	0.2654	0.1649	273.4

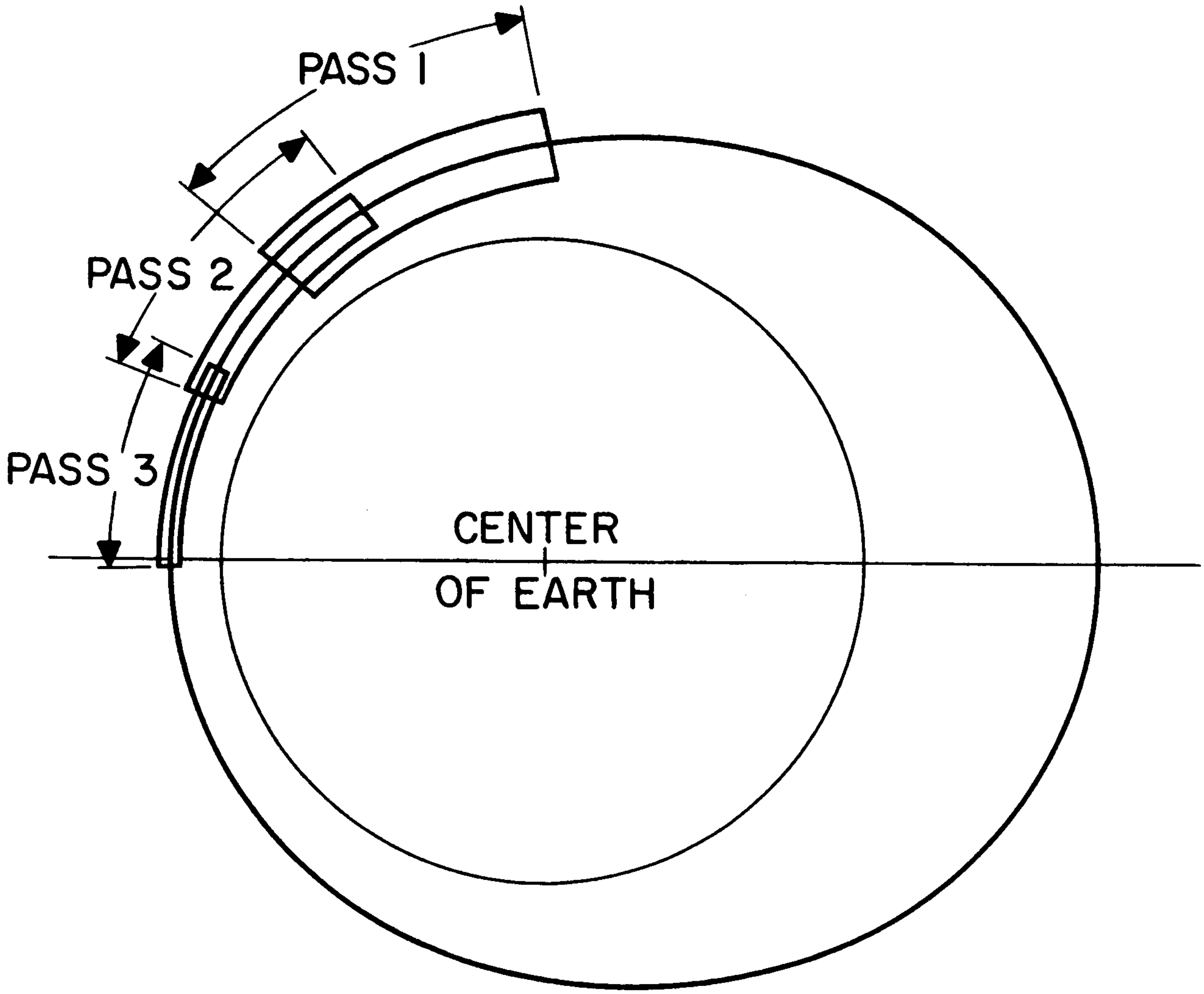

Fig. 1. Orbit of 1958 Beta II

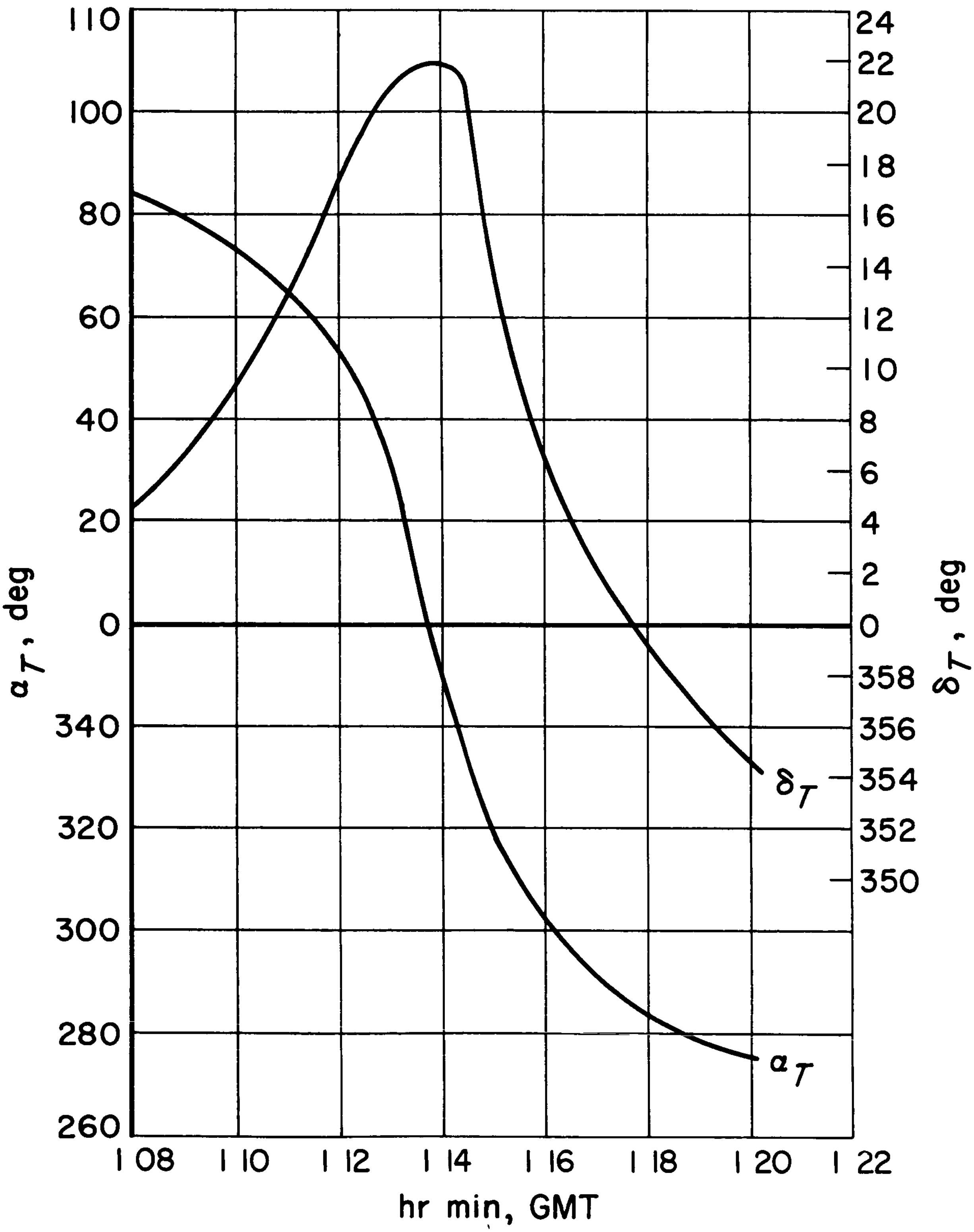

Fig. 2. Hour Angle and Declination Angle of the True Orbit of 1958 Beta II

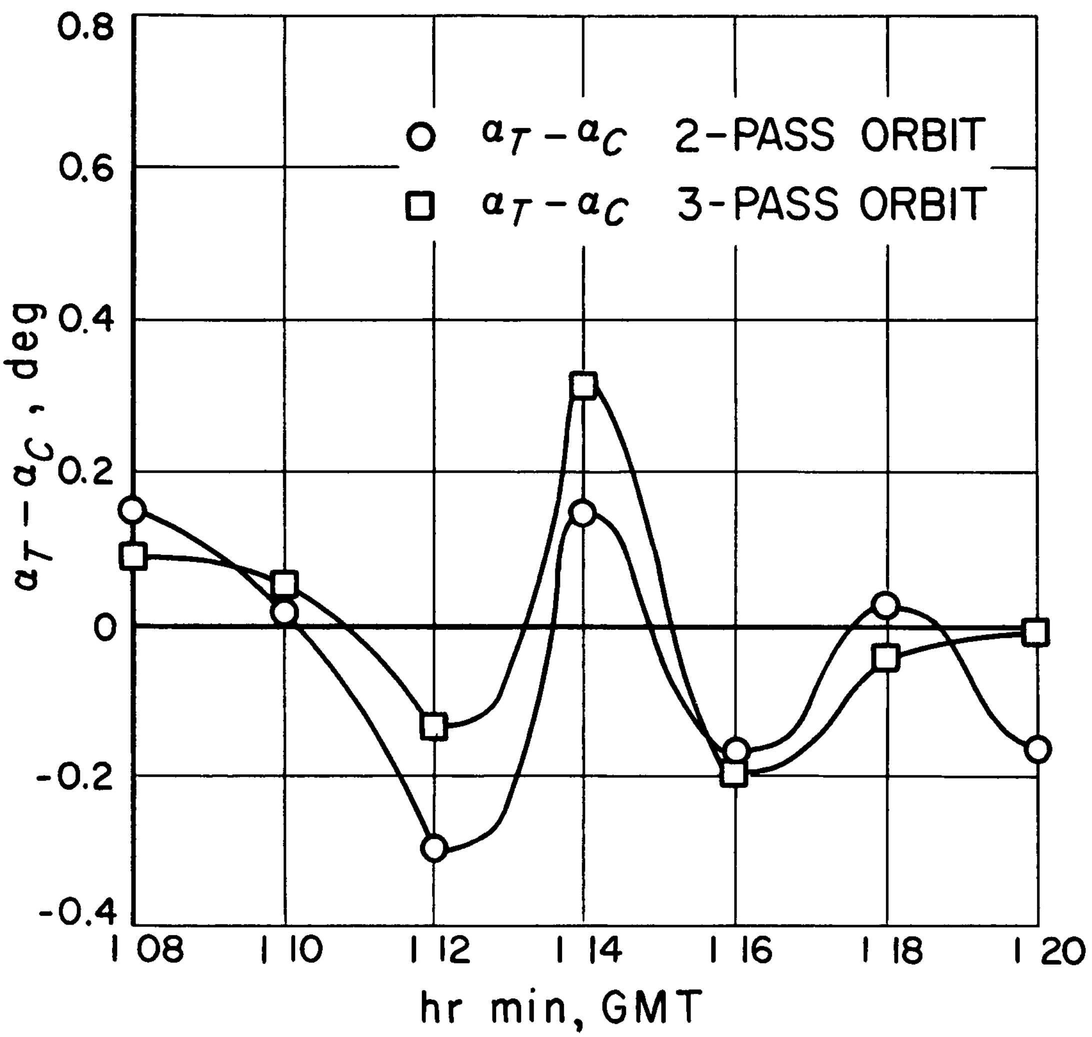

Fig. 3. Prediction Error in Hour Angle; Angle Data Only

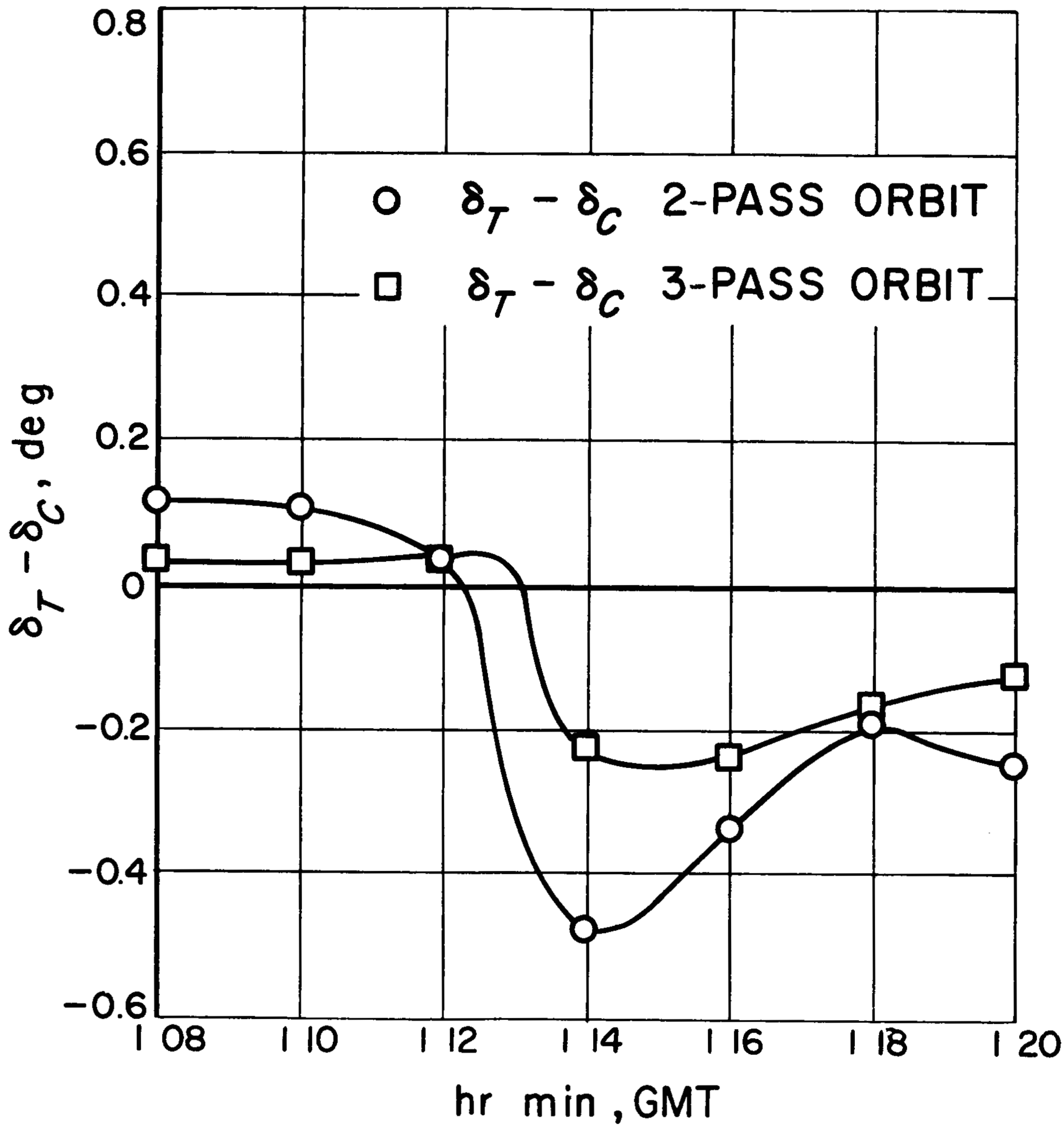

Fig. 4. Prediction Error in Declination Angle; Angle Data Only

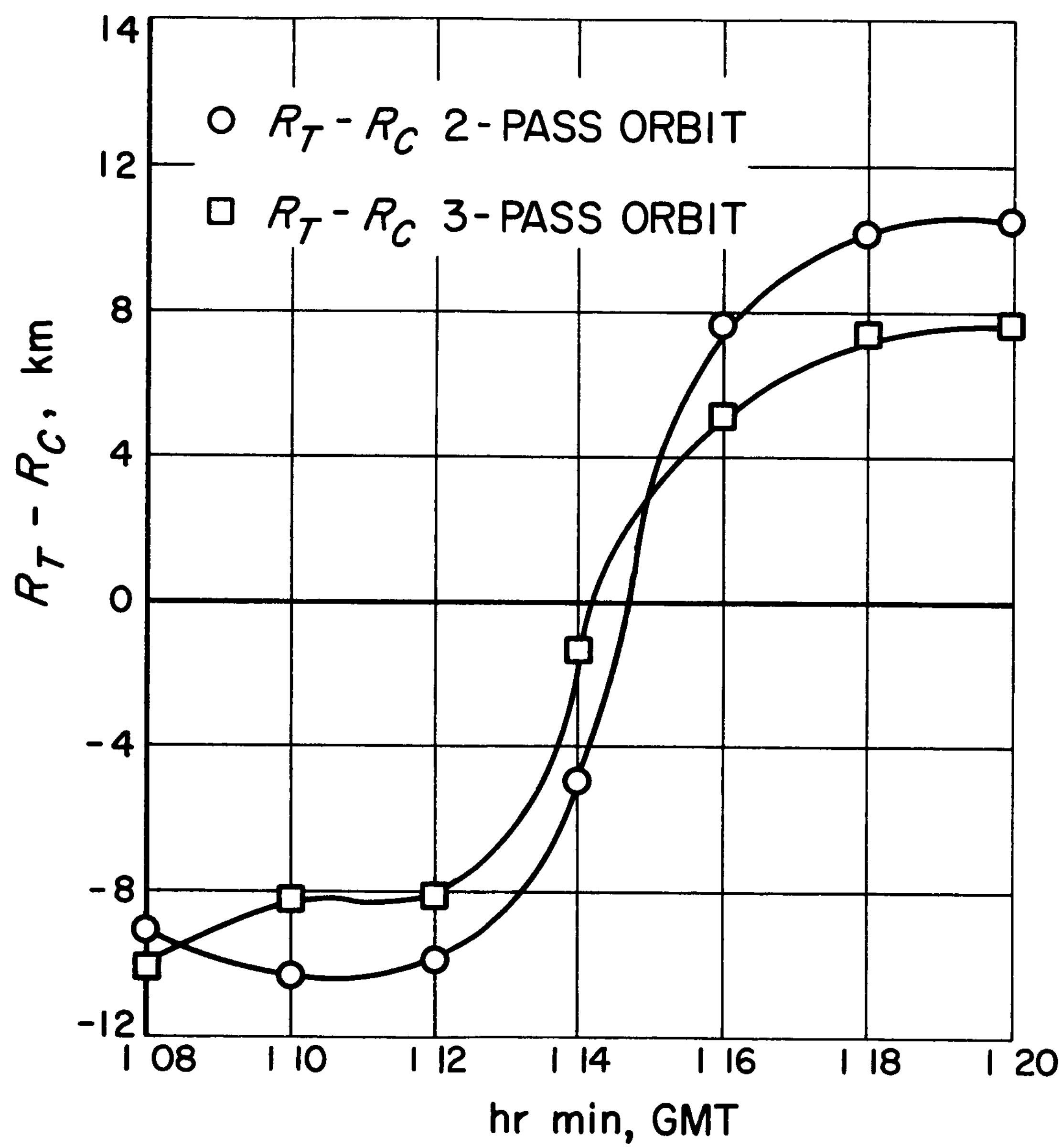

Fig. 5. Prediction Error in Slant Range; Angle Data Only

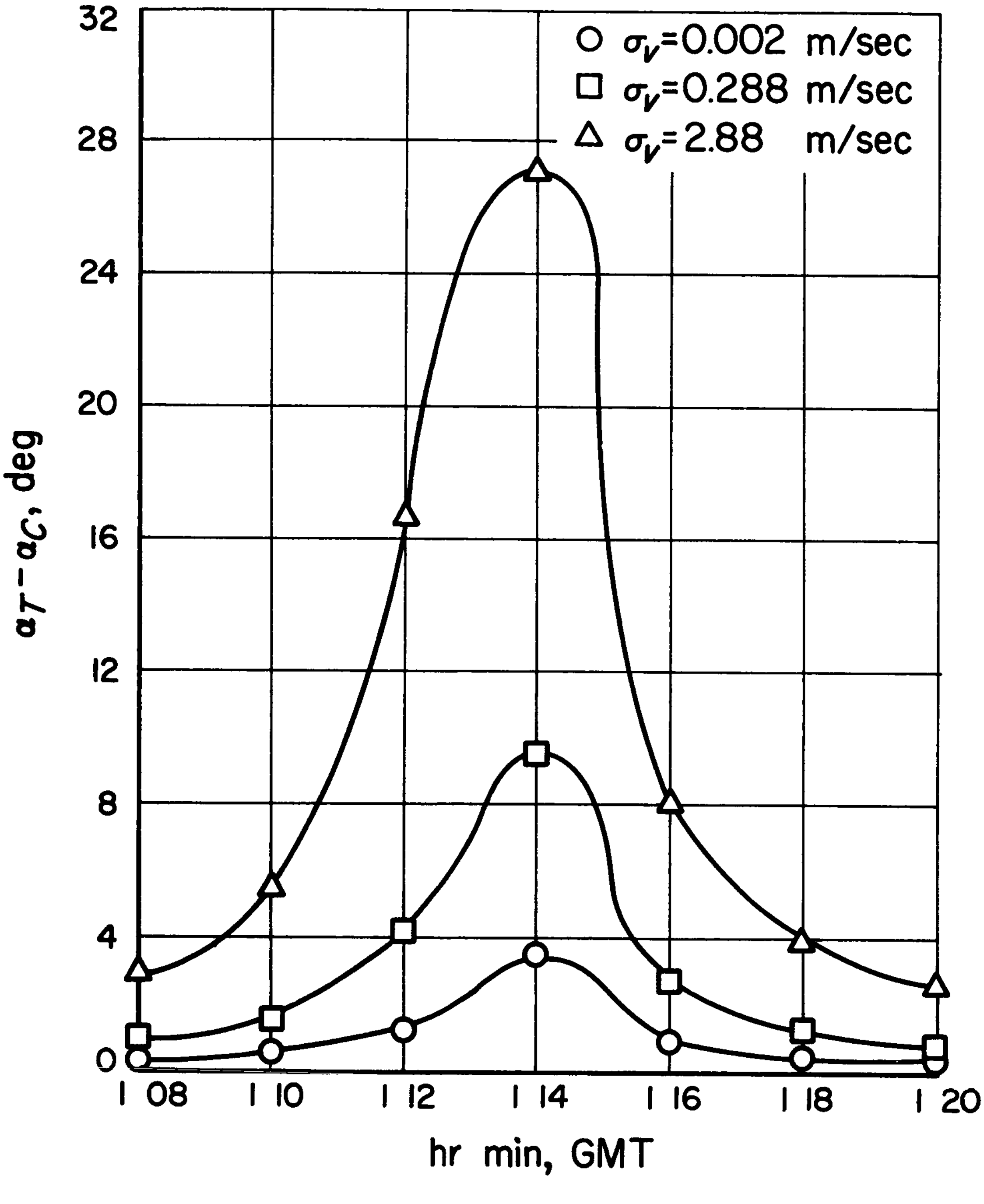

Fig. 6. Prediction Error in Hour Angle; Orbit Determined With First Pass and Completed Velocity Data

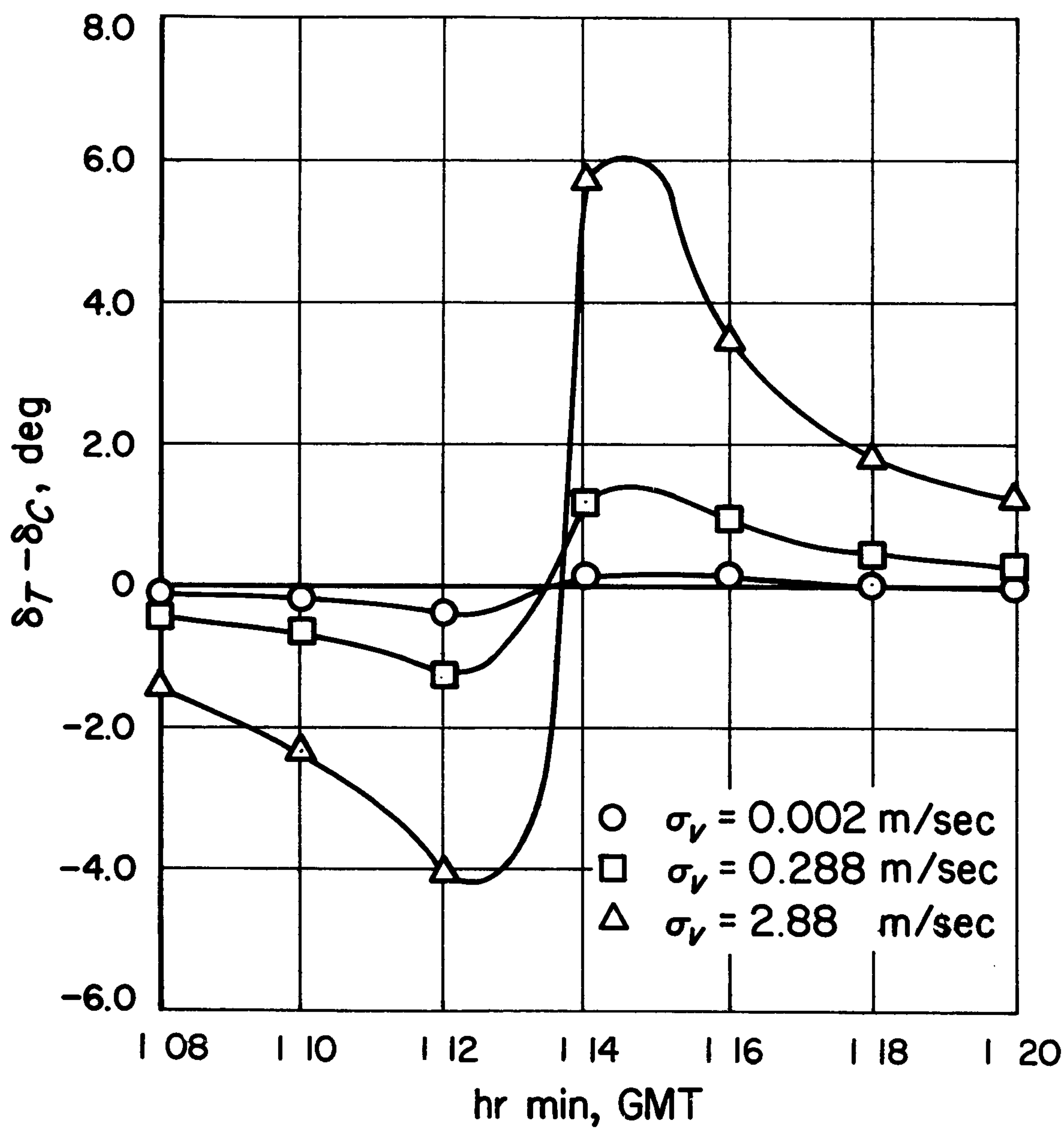

Fig. 7. Prediction Error in Declination Angle; Orbit Determined With First Pass and Computed Velocity Data

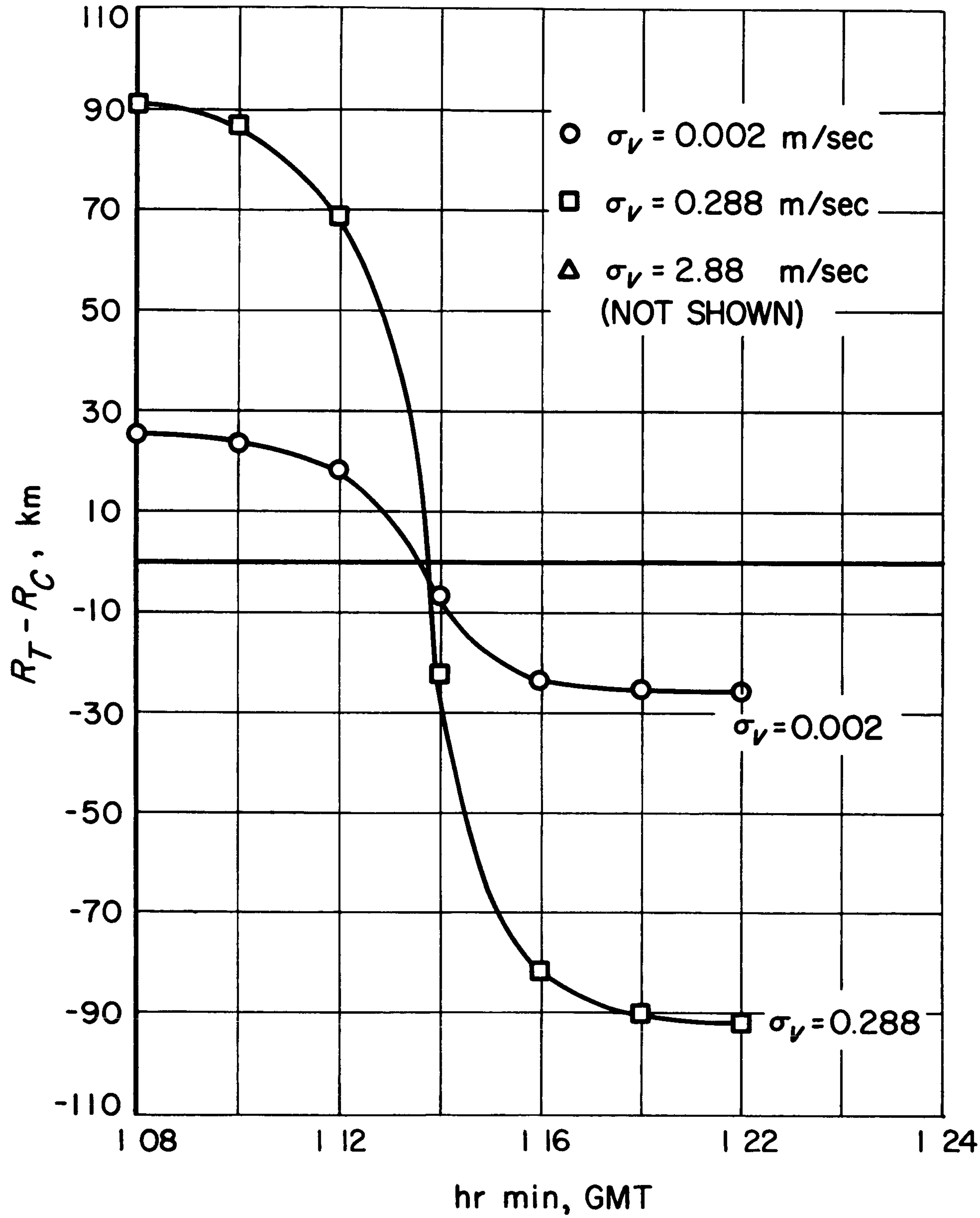

Fig. 8. Prediction Error in Slant Range; Orbit Determined With First Pass and Computed Velocity Data

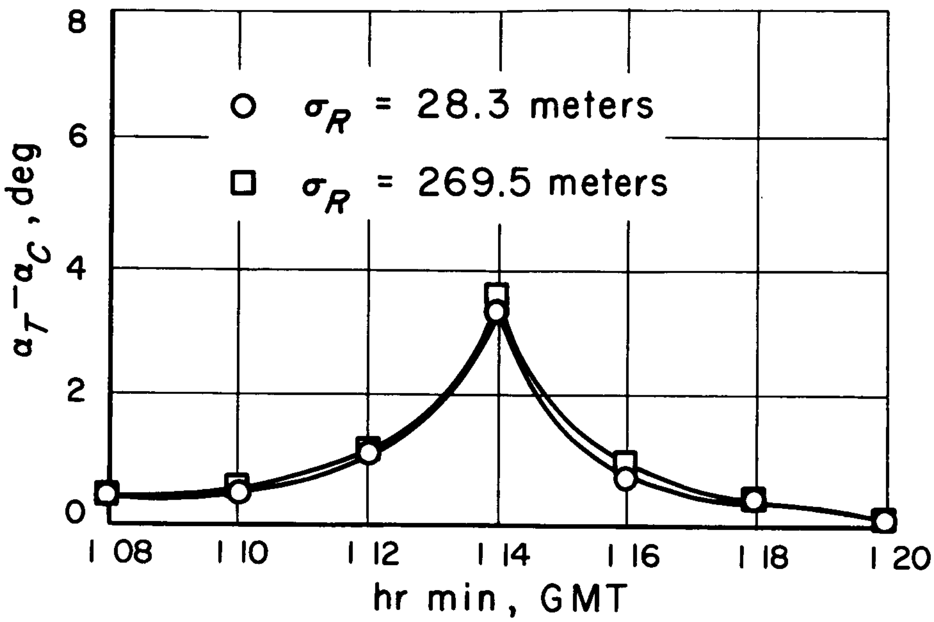

Fig. 9. Prediction Error in Slant Range; Orbit Determined With First Pass and Computed Slant Range Data

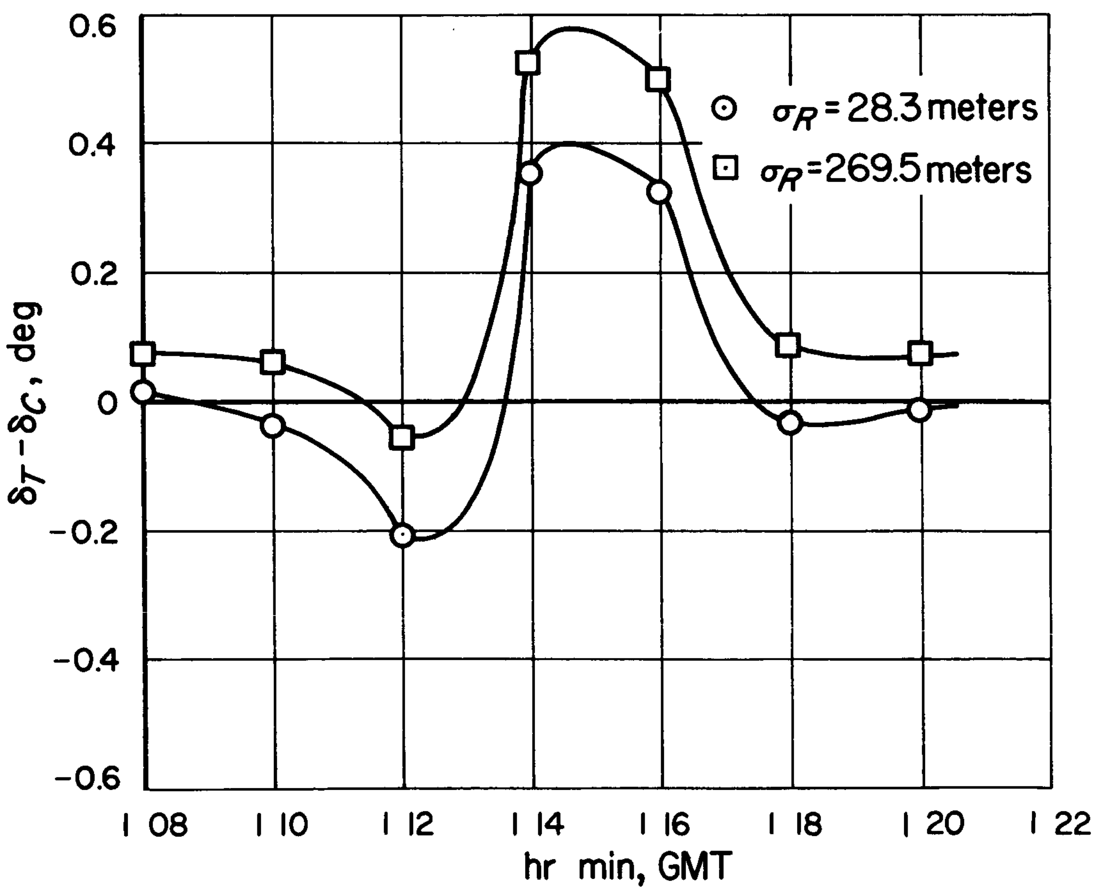

Fig. 10. Prediction Error in Declination Angle; Orbit Determined With First Pass and Computed Slant Range Data

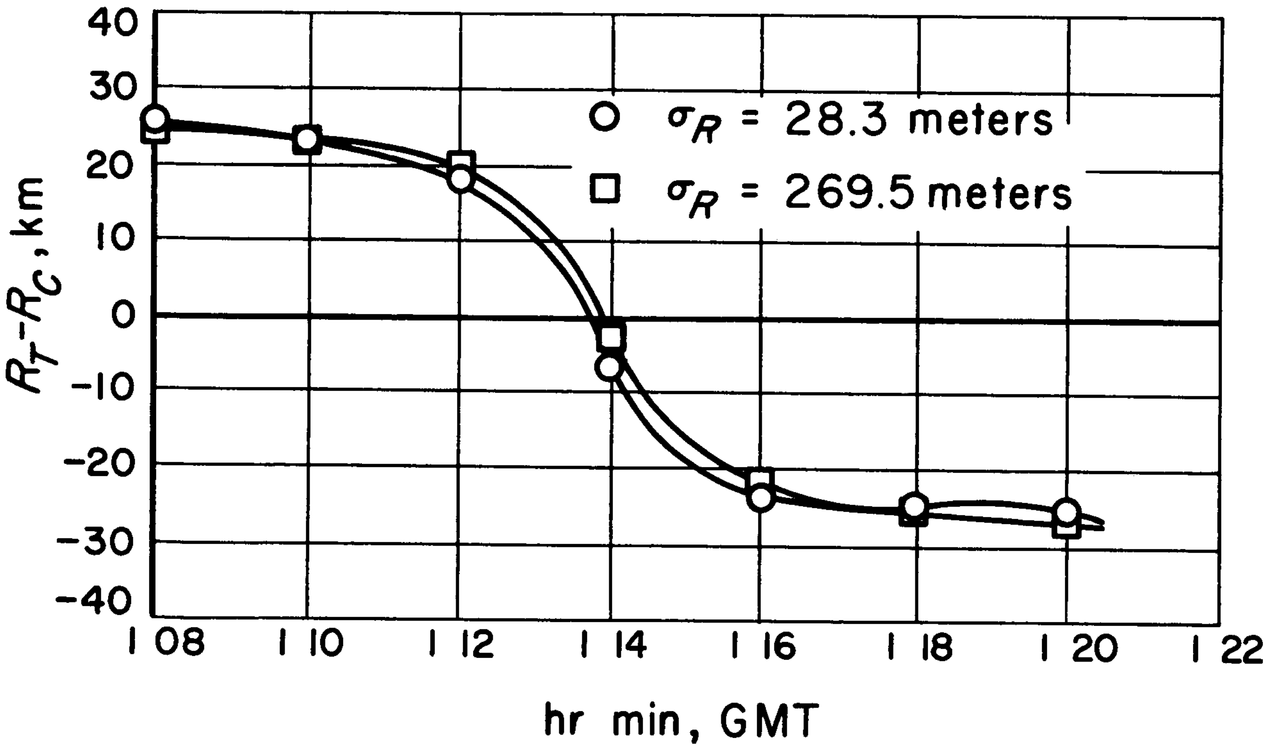

Fig. 11. Prediction Error in Slant Range; Orbit Determined With First Pass and Computed Slant Range Data